Prob. Set 6

Some functions have asymptotes that are neither horizontal nor vertical, but some other line. Such asymptotes are somewhat more difficult to identify and we will ignore them.

If the domain of the function does not extend out to infinity, we should also ask what happens as x approaches the boundary of the domain. For example, the function $y = f(x) = 1/\sqrt{r^2 - x^2}$ has domain $-r < x < r$, and y becomes infinite as x approaches either r or $-r$. In this case we might also identify this behavior because when $x = \pm r$ the denominator of the function is zero.

If there are any points where the derivative fails to exist (a cusp or corner), then we should take special note of what the function does at such a point.

Finally, it is worthwhile to notice any symmetry. A function $f(x)$ that has the same value for $-x$ as for x, i.e., $f(-x) = f(x)$, is called an "even function." Its graph is symmetric with respect to the y-axis. Some examples of even functions are: x^n when n is an even number, $\cos x$, and $\sin^2 x$. On the other hand, a function that satisfies the property $f(-x) = -f(x)$ is called an "odd function." Its graph is symmetric with respect to the origin. Some examples of odd functions are: x^n when n is an odd number, $\sin x$, and $\tan x$. Of course, most functions are neither even nor odd, and do not have any particular symmetry.

ooh la la

We will do more!

Exercises 5.5.

Sketch the curves. Identify clearly any interesting ~~features, including local maximum and minimum points, inflection points, asymptotes, and~~ intercepts.

1. $y = x^5 - 5x^4 + 5x^3$

2. $y = x^3 - 3x^2 - 9x + 5$

3. $y = (x-1)^2(x+3)^{2/3}$

4. $x^2 + x^2y^2 = a^2y^2$, $a > 0$.

5. $y = xe^x$

6. $y = (e^x + e^{-x})/2$

7. $y = e^{-x}\cos x$

8. $y = e^x - \sin x$

9. $y = e^x/x$

10. $y = 4x + \sqrt{1-x}$

11. $y = (x+1)/\sqrt{5x^2+35}$

12. $y = x^5 - x$

13. $y = 6x + \sin 3x$

14. $y = x + 1/x$

15. $y = x^2 + 1/x$

16. $y = (x+5)^{1/4}$

17. $y = \tan^2 x$

18. $y = \cos^2 x - \sin^2 x$

19. $y = \sin^3 x$

20. $y = x(x^2+1)$

21. $y = x^3 + 6x^2 + 9x$

22. $y = x/(x^2 - 9)$

23. $y = x^2/(x^2+9)$

24. $y = 2\sqrt{x} - x$

25. $y = 3\sin(x) - \sin^3(x)$, for $x \in [0, 2\pi]$

26. $y = (x-1)/(x^2)$

For each of the following five functions, identify any vertical and horizontal asymptotes, and identify intervals on which the function is concave up and increasing; concave up and decreasing; concave down and increasing; concave down and decreasing.

Prob. #	Problem	Work/Final answer circled

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